

CONFIDENTIAL*

**MATHEMATICS S
PAPER 1**
**MATHEMATICS T
PAPER 1**

Three hours

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**PEPERIKSAAN PERCUBAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
NEGERI PAHANG DARUL MAKMUR 2012**

Instructions to candidates:

Answer all questions.

Answers may be written in either English or Bahasa Malaysia.

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures,

or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae and graph paper are provided.

This question paper consists of 5 printed pages.

Mathematical Formulae for Paper 1 Mathematics T / Mathematics S :

Logarithms :	Integration :
$\log_a x = \frac{\log_b x}{\log_b a}$	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
Series :	
$\sum_{r=1}^n r = \frac{1}{2} n(n+1)$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$
$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$	$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
$\sum_{r=1}^n r^3 = \frac{1}{4} n^2 (n+1)^2$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

Series:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n, \text{ where } n \in \mathbb{N}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots + \frac{n(n-1)\cdots(n-r+1)}{r!} x^r + \dots, \text{ where } |x| < 1$$

Coordinate Geometry :

The coordinates of the point which divides the line joining (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ is

$$\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right)$$

The distance from (x_1, y_1) to $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Numerical Methods :

Newton-Raphson iteration for $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Trigonometry :

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

Trapezium rule :

$$\int_a^b f(x) dx \approx \frac{1}{2} h [y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$$

$$\text{where } y_r = f(a + rh) \text{ and } h = \frac{b-a}{n}$$

1. Solve the following equations:

$$\log_4 x + \log_x 4 = 2.5 \quad [5 \text{ marks}]$$

2. If $y = \frac{3\cos x}{x}$, prove that $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + xy = 0$. [5 marks]

3. In a geometric progression, the third term is 6 and the sixth term is $-\frac{2}{9}$. Find the first term and the common ratio of the series. [2 marks]
Hence, find :

- (a) S_n , the sum of the first n terms of the series. [2 marks]
(b) S_∞ , the sum to the infinity of the series. [2 marks]

4. Express $f(x) = \frac{1-4x-x^2}{(x+2)(x^2+1)}$. in partial fractions.

$$\text{Hence, show that } \int_0^1 f(x)dx = \ln \frac{3}{4} \quad [6 \text{ marks}]$$

5. Using the laws of the algebra of sets, show that

$$(A - B) \cap (A \cap B)' = A \cap B' \quad [7 \text{ marks}]$$

- 6.(a) Find all the values of k that satisfy the inequality $\frac{1-2k}{3k^2-(1-2k)} \geq 0$. [3 marks]

- (b) Solve the inequality $\left| \frac{x+1}{3x-5} \right| < 1$. Give your answer in the interval form. [4 marks]

7. Given that $y^2 = \ln(x^2 y)$ where $x > 0$ and $y > 0$,

- (a) show that $\frac{dy}{dx} = \frac{2y}{x(2y^2-1)}$ [3 marks]

- (b) find the small change in x , correct to three decimal places, when the value of y changes from 1 to 1.01. [4 marks]

8. Expand $\left(\frac{1-x}{1+2x} \right)^{\frac{1}{2}}$ in ascending powers of x up to and including the term x^2 .

[4 marks]

By finding the range of the values of x where the expansion is valid, explain why $x = \frac{1}{4}$ can be used in the above expansion to estimate the value of $\sqrt{2}$. [2 marks]

Hence, estimate the value of $\sqrt{2}$ correct to four decimal places. [2 marks]

- 9.(a) The polynomial $2x^3 - 3ax^2 + ax + b$ has the factor $x - 1$ and leaves a remainder of -54 when divided by $x + 2$. Find the values of a and b .

Using these values of a and b , factorise the above polynomial completely.

Subsequently, find all the real zeroes of the polynomial $2x^6 - 9x^4 + 3x^2 + 4$. [6 marks]

- (b) Given $y = \frac{3x - 9}{(x + 1)(x - 2)}$, show that y does not have any real value between $\frac{1}{3}$ and 3 for all real values of x . [4 marks]

10. A parabola has parametric equations $x = t(t - 2)$ and $y = 2(t - 1)$.

- (a) By finding the Cartesian equation of the parabola, determine
 (i) the vertex,
 (ii) the focus
 (iii) the equation of the directrix [5 marks]

- (b) Show that the equation of the normal to the parabola at the point when $y = 4$ is $\frac{x}{5} + \frac{y}{10} = 1$. [5 marks]

- (c) Calculate the shortest distance between the vertex of the parabola and the normal to the parabola at the point when $y = 4$. [2 marks]

11. The adjoint of matrix $A = \begin{pmatrix} 0 & 2 & -1 \\ 0 & 8 & -1 \\ 4 & 10 & -1 \end{pmatrix}$ is $\begin{pmatrix} 2 & -8 & 6 \\ -4 & 4 & 0 \\ 32 & k & 0 \end{pmatrix}$.

- (a) Show that matrix A is non-singular (i.e. A is invertible). [3 marks]
 (b) Find
 (i) the value of k ,
 (ii) the inverse of matrix A . [2 marks] [2 marks]

The matrix $\begin{pmatrix} 3 & 1 & 4a+10b \\ 2b-c & 0 & 8b-c \\ 29+c & 16 & 1 \end{pmatrix}$ where a, b and c are constants, is a symmetric matrix.

- (c) Write a system of equations in terms of a, b and c .
 (d) Hence, find the values of a, b and c using matrix.

[2 marks]
 [4 marks]

- 12(a) The continuous function $f(x)$ is defined by

$$f(x) = \begin{cases} \frac{2(x-1)}{|x-1|}, & x < 1 \\ 4x-k, & x \geq 1 \text{ and } k \text{ is a constant} \end{cases}$$

- Find : (i) the value of $\lim_{x \rightarrow 1^-} f(x)$,
 (ii) the value of k .

[2 marks]
 [2 marks]

- (b) The functions f and g are defined by :

$$\begin{aligned} f : x \rightarrow \frac{1}{1+x^2}, & \quad x < 0 \\ g : x \rightarrow -\sqrt{1-x^2}, & \quad -1 \leq x \leq 1 \end{aligned}$$

- (i) By using sketch graphs of the functions f and g on the same axes, determine the range of function f and function g .
 (ii) Determine whether the function $f \circ g$ exists.
 (iii) Explain why the function f has an inverse function, f^{-1} .
 Find the inverse function, f^{-1} in the same form as given for f

[5 marks]
 [2 marks]
 [3 marks]

END OF QUESTION PAPER

PEPERIKSAAN PERCUBAAN

SIJIL TINGGI PERSEKOLAHAN MALAYSIA

NEGERI PAHANG DARUL MAKMUR 2012

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1. The length y meters of a cable subjected to a load of x kilogram. The values of y were measured for each of 15 values of x . The following quantities were calculated from 15 pairs of values.

$$\Sigma x = 225, \quad \Sigma y = 238, \quad \Sigma x^2 = 3625, \quad \Sigma y^2 = 3792, \quad \Sigma xy = 3581$$

- (a) Calculate the Pearson's correlation coefficient for the length of load. [3 marks]
 (b) Calculate the coefficient of determinant. Comment on your answer. [3 marks]

2. 500 lecturers attended a seminar concerning the standard of higher education in our country and 120 agreed that the standard of higher education is high.

- (a) Obtain a 95% confidence interval for the proportion of lecturers who agreed that the standard of higher education in our country is high. [4 marks]
 (b) Find the smallest sample size which needs to be taken or considered in order that the error of estimation is not more than 0.05 at the 90% confidence interval. [5 marks]

3. X is a discrete random variable with probability function as follow:

$$f(x) = \begin{cases} \frac{bx}{4} & , x = 1, 2, 3 \\ \frac{x}{21} & , x = 4, 5 \\ 0 & , \text{otherwise} \end{cases}$$

where b is a constant.

- (a) Find the value of b . [2 marks]
 (b) Sketch a graph of $f(x)$ [2 marks]
 (c) Find the mean and standard deviation of X . [4 marks]

4. The time taken in minutes by exam candidates in an examination is normally distributed.

A random sample of 200 students is taken and the time x taken by each student in completing the examination is recorded. The results are summarized as follows:

$$\Sigma x = 16000 \quad \Sigma x^2 = 1283184$$

- (a) Calculate the unbiased estimate for the population mean μ and the population variance, σ^2 . [3 marks]
 (b) If 40 of the students who sat for the examination did not complete the test, calculate a 95% confidence interval for the population proportion of students who completed the examination. [4 marks]
 (c) Teacher Tan claims that 80% of the exam candidates managed to complete the examination with a 95% confidence interval. Comment on the teacher's claim. [2 marks]

5. A and B are two events such that $P(A) = \frac{1}{4}$, $P(A|B) = \frac{3}{5}$, $P(A|B') = \frac{1}{7}$

(a) Show that $P(A \cap B) = \frac{9}{64}$ [4 marks]

(b) Find the value of

(i) $P(B)$ [2 marks]

(ii) $P(B|A)$ [2 marks]

(c) State whether events A and B are independent. Give a reason for your answer.

[2 marks]

6. The diameter of a type of straws has normal distribution with mean μ mm and standard deviation σ mm. Given that 10% of the straws have diameters more than 18.533 mm and 95% of the straws are with diameters at least 17.632 mm.

(a) Find the values of μ and σ correct to 3 decimal places. [5 marks]

(b) Calculate the probability that the mean diameter of a random sample of 9 screws is between 17.85 mm and 18.05 mm. [4 marks]

7. The table below shows the income tax collected (in million RM) for each quarter and the centered moving averages.

Year	Quarter	Income tax (in million RM)	Centered moving average
2005	1	15	
	2	28	
	3	34	22.875
	4	16	21.75
2006	1	12	21.125
	2	22	21.75
	3	35	22.5
	4	20	23.25
2007	1	14	23.375
	2	26	23.5
	3	32	
	4	24	

Based on multiplicative model, calculate the adjusted seasonal variation for each of the four quarters. [4 marks]

8. The table below shows the duration in minutes taken by 200 students to complete a Mathematics Problem in a competition.

Duration (minutes)	Frequency
0.0 - 2.9	25
3.0 – 5.9	48
6.0 – 8.9	56
9.0 – 11.9	24
12.0 – 14.9	20
15.0 – 17.9	17
18.0 – 20.9	10

- (a) Draw a histogram and comment on the skewness of the distribution. [3 marks]
 (b) Calculate the mean and standard deviation. [4 marks]

9. The relationship between the revision time, x hours per week and the examination marks, y of eight students for Mathematics S are given in the following table:

Revision time per week, x hours	12	11	10	8	7	6	5	4
Examination marks, y	65	70	60	52	50	33	40	35

- (a) Plot a scatter diagram for the above data. [2 marks]
 (b) Find the regression line of y on x in the form of $y = a + bx$ where a and b are expressed correct to 2 decimal places. Draw the graph of the regression line on your scatter diagram. [7 marks]
 (c) Find the score, to the nearest integer, of a student who spent 8.5 hours per week on revision. [2 marks]

10. The following table shows the number of handphone sold in a departmental store for the year 2009 and 2010.

Type of handphone	2009		2010	
	Price (RM)	Quantity	Price (RM)	Quantity
A	130	26	195	40
B	230	18	270	38
C	380	19	420	21
D	580	30	720	25

By using 2009 as the base year,

- (a) Calculate the average of relative quantity index of handphones A, B, C and D for the year 2010. [3 marks]
- (b) Calculate the Paasche price index for the year 2010 and comment on your answer. [3 marks]

11. The following table shows the activities for a project and their preceding activities and duration.

Activity	Preceding activity	Duration (weeks)
A	-	2
B	-	3
C	-	6
D	A	9
E	B	5
F	C	9
G	B, D	5
H	F	4

- (a) Draw a network diagram to for the above project. [2 marks]
- (b) Calculate the total float for each activity. Hence, determine the critical path and the minimum completion time of the project. [7 marks]
- (c) If activity E is delayed for 8 weeks, is there any effect on the completion time of the project? Give a reason for your answer. [2 marks]

12. A furniture manufacturer produces tables, sofas and chairs. The profits per item are RM50, RM60, RM20 respectively.

The following table shows the initial tableau for the maximization of total profit, P.

The variable X_1 , X_2 , and X_3 represent the number of tables, sofas and chairs to be produced per week.

Basic	P	X_1	X_2	X_3	S_1	S_2	S_3	Solution
P	1	-50	-60	-20	0	0	0	0
S_1	0	5	10	3	1	0	0	500
S_2	0	1	3	1	0	1	0	200
S_3	0	2	0	1	0	0	1	180

- (a) Based on the initial tableau given, formulate the linear programming model to determine the number of tables, sofas and chairs that should be produced per week in order to maximize the total profit. [4 marks]
- (b) Construct the subsequent tableaus until the optimal solution is obtained. [4 marks]
- (c) Hence, state the number of tables, sofas and chairs that should be produced per week in order to maximise the total profit. State also the maximum profit. [2 marks]

Mathematical Formulae for Paper 2 Mathematics S :

Correlation and regression

Pearson correlation coefficient:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 - \sum(y_i - \bar{y})^2}}$$

Regression line y on x : $y = a + bx$

$$\text{Where } b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}, \quad a = \bar{y} - b\bar{x}$$